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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2017

Second Semester

Core Course 2—ANALYTIC GEOMETRY, TRIGONOMETRY AND MATRICES

(Common for B.Sc. Mathematics Model I, Model II and B.Sc. Computer Application)

(2013 Admission onwards)

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. What is the point of intersection of the tangents at t_1 and t_2 on the parabola $y^2 = 4ax$?
2. How many normals can be drawn from a given point to a parabola ?
3. Define director circle of an ellipse.
4. What is the condition for the line $lx + my + n = 0$ to touch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$?
5. What is a rectangular hyperbola ?
6. What is the equation of the tangent at a point θ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$?
7. Prove that $\cosh 2\theta = 1 + 2\sinh^2\theta$.
8. Separate $\text{Co}(\alpha + i\beta)$ into real imaginary parts.
9. Define rank of a matrix.
10. State Cayley-Hamilton theorem.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Define a conjugate hyperbola.
12. Find the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 17x + y + 14 = 0$.
13. Show that if $\alpha - \beta$ is a constant, the chord joining the points α and β on an ellipse touches a fixed ellipse.

Turn over

14. What is the pole of the line $lx + my + n = 0$ with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

15. Show that in a parabola if the normal at P meets the axis in G, then SG = SP, where S is the focus.

16. If the normal at the point f_1 on a parabola $y^2 = 4ax$ meet it again at t_2. Prove that $t_2 = -t_1 - \frac{2}{t_1}$.

17. Find the point of intersection of the tangents at θ and ϕ on the ellipse $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

18. If the normal at a point P on the ellipse meets the major axis at G, then show that SG = C·SP.

19. Find the condition in order that the line $\frac{l}{r} = A \cos B + B \sin \theta$ may be a tangent to the conic $\frac{l}{r} = 1 + c \cos \theta$.

20. Find the rank of the matrix $\begin{bmatrix} 2 & 4 & 3 & 2 \\ 3 & 3 & 1 & 4 \end{bmatrix}$.

21. Find the characteristic equation of the matrix $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$.

22. If $\tan \frac{\theta}{2} = \tanh \frac{u}{2}$, show that $u = \log \tan \left[\frac{\pi}{4} + \frac{\theta}{2}\right]$.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 marks.

23. Separate $\log (\alpha + i\beta)$ into real and imaginary parts.

24. If $\sin (\alpha + i\beta) = \text{Cos } \theta + i \sin \theta$, prove that $\cos^2 \alpha = \sin h^2 \beta$.

25. Show that $\frac{c \sin \theta}{1!} + \frac{c^3 \sin 3\theta}{3!} + \frac{c^5 \sin 5\theta}{5!} + - - -$

$= \sin (c \sin \theta) \cosh (c \cos \theta)$.

26. Sum to infinity $\sinh \alpha - \frac{1}{2} \sinh 2\alpha + \frac{1}{3} \sinh 3\alpha \; - - - \infty$.

27. Find the asymptotes of the conic $\frac{l}{r} = 1 + e \cos \theta$.

28. Obtain the polar equation of a circle.

29. Prove that the chords of a rectangular hyperbola which subtend a right angle at a focus touch a fixed parabola.

30. Using Cayle-Hamilton theorem, show that $A^3 - 6A^2 + 11A - 6I = 0$, where $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$ and hence find A^{-1}.

31. Test for consistency and solve the system of equations

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6.$$

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (i) Prove that a circle will cut a parabola in four points and the algebraic sum of the ordinates of the four points is zero.

(ii) If P and D are extremities of conjugate diameters of the ellipse. Show that the locus of the point of intersection of the tangents at P and D is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.

33. (i) Find the locus of the feet of the parpendiculars from the centre on normals to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

(ii) Prove that the product of the perpendiculars drawn from any point on a hyperbola to its asymptotates is constant.

34. (i) Factorize $x^8 + 1$ into real factors.

(ii) Sum the series $1 + \frac{1}{2}\cos^2\theta - \frac{1}{2.4}\cos 4\theta + \frac{1.3}{2.4.6}\cos 6\theta \ldots$. where $-\frac{\pi}{2} c\theta \frac{c\pi}{2}$.

35. (i) Solve by Cramers rule

$$x + y + z = 5$$
$$x - 2y - 3z = -1$$
$$2x + y - z = 3$$

(ii) Determine the eigen vectors of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

(2 × 15 = 30)